An introduction to ocean circulation models and modelling

Lectures given at
ADVANCED SCHOOL ON EARTH SYSTEM MODELLING, IITM Pune
& Department of Mathematics , Savitribai Phule Pune University
Pune, India
July 18-29, 2016

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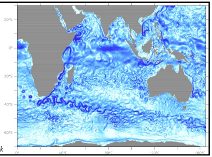
NOAA/GFDL/Princeton University

$$
\begin{aligned}
[\partial_t + (f + \mathcal{M})\,\hat{\mathbf{z}} \wedge]\,(\mathrm{d}z\,\rho\,\mathbf{u}) = {} & \rho\,\mathrm{d}z\,\mathcal{S}^{(\mathbf{u})} - \nabla_s \cdot [\,\mathrm{d}z\,\mathbf{u}\,(\rho\,\mathbf{u})] \\
& - \mathrm{d}z\,(\nabla_s\,p + \rho\,\nabla_s\Phi) + \mathrm{d}z\,\rho\,\mathbf{F} \\
& - [\,\rho\,(w^{(z)}\,\mathbf{u} - \kappa\,\mathbf{u}_{,z})\,]_{s=s_{k-1}} \\
& + [\,\rho\,(w^{(z)}\,\mathbf{u} - \kappa\,\mathbf{u}_{,z})\,]_{s=s_k}
\end{aligned}
$$

$$
\partial_t(\mathrm{d}z\,\rho\,C) = \mathrm{d}z\,\rho\,\mathcal{S}^{(C)} - \nabla_s \cdot [\mathrm{d}z\,\rho\,(\mathbf{u}\,C + \mathbf{F})] - [\rho\,(w^{(z)}\,C + F^{(z)})]_{s=s_{k-1}} + [\rho\,(w^{(z)}\,C + F^{(z)})]_{s=s_k}
$$



Outline

Bangalore Oct2004 and Hyderabad Aug2013

Thanks for bringing me back!

Outline

Two pillars of numerical oceanography

- OCEAN MODELS:
  - Repository for mechanistic theories of how the ocean works, with numerical methods transforming theories into a computational tool for scientific investigations.
  - Mathematically formulated physical theories and numerical methods provide the foundation.
- OCEAN MODELLING:
  - The use of numerical simulations as an experimental tool to help deduce mechanisms for emergent space-time patterns of ocean phenomena.
  - Math physical theories and analysis methods provide the foundation.

Useful distinction

It is useful to distinguish the reductionist science forming the foundation of *ocean models* from the emergent science arising from the process of *ocean modelling*.

Topics to be discussed

- Science of ocean models
  - continuum thermo-hydrodynamical equations
  - algorithms for hydrostatic primitive equations
  - subgrid scale parameterizations
- Tutorial on sea level analysis
  - equations of sea level
  - Public lecture on sea level in a changing climate
- I expect to merely whet your appetite for further study (and hopefully make you less afraid of ocean models and modelling!
- My personal experience is with the physics and numerics of large-scale ocean climate circulation models, and this experience will bias my focus.

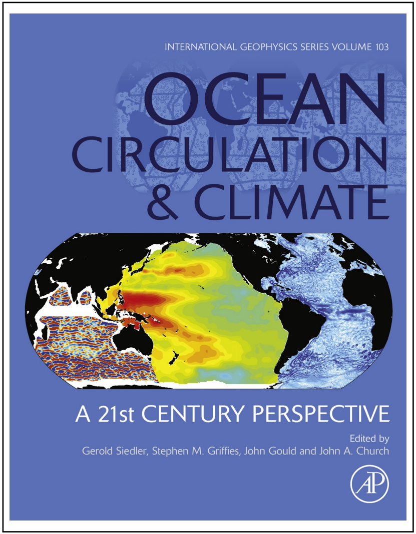
Caution to the student

There will be mathematics! It will be at a level no higher than vector calculus and partial differential equations (no tensor analysis here ☺).

Published background for these lectures

- Science of ocean models
  - Griffies and Treguier (2013): chapter from 2nd edition of **Ocean Circulation and Climate**
  - Griffies (2009): chapter in **Encyclopedia of Ocean Sciences**
  - Griffies and Adcroft (2008): chapter discussing the formulation of ocean model equations from an AGU monograph.
  - Griffies (2005): “Some ocean model fundamentals”
  - Griffies (2004): monograph on ocean climate model fundamentals
- Ocean models and ocean modelling
  - Griffies et al. (2000): review of ocean climate model development
  - Griffies et al (2009): research article on Coordinated-Ocean ice Reference Experiments (COREs)
  - Griffies et al (2010): White Paper from the OceanObs09 conference summarizing the status of ocean models
- Science emerging from the practice of ocean modelling: see nearly any paper in *Ocean Modelling*, *Journal of Physical Oceanography*, *Journal of Climate*, etc.

Ocean Circulation and Climate, 2nd Edition (2013)



General motivational comments

- Ocean model fundamentals and the use of ocean models as a tool for science involves some of the most difficult problems in classical and computational physics.
  - turbulence closures and subgrid scale parameterizations
  - analysis and rationalization of massive datasets
  - efficient methods for discretizing continuous media.
- We are also touching on elements of the most important environmental and societal problem facing the planet.
  - Climate warming is happening and humans are the key reason.
  - The ocean's role in the earth climate is significant.
  - Providing rational and robust models for understanding and predicting climate is a central element of oceanography and climate science.
  - Now is an incredibly exciting time to enter this field, particularly for those who feel passionate about diving deep into some of the most difficult intellectual problems in science while addressing some of the most important problems for the biosphere.

Some words of encouragement

Our job is to transfer knowledge that is in the head of teachers into your head. Your help is needed with this communication problem ☺.

- You will be challenged.
  - Ask questions whenever they arise. Do not be shy!
  - The material at this school is far more than can fit into one head within a 10 day course.
  - Select that which is useful to you. Some ideas may be useful now, some in a few years, some never.
- Do not despair if you are totally lost, even if everyone else fully understands ☺.
  - Much time (years!) may be needed to penetrate the ideas and details.
  - It may be sufficient for some students to just understand the main points.
  - But please do push yourself a bit more than normal. These courses offer valuable and somewhat rare learning opportunities.

Outline

2 Motivation for using ocean models

Why use ocean models?

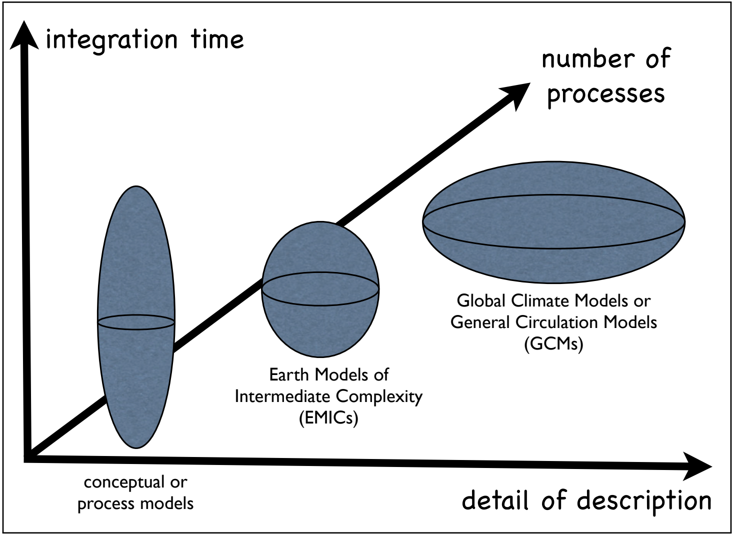
Ocean models are ubiquitous in ocean/climate science. Why?

- Numerical models are the primary means available for probing, in a nearly controlled manner, the ocean/climate system.
  - There is one natural ocean, yet many numerical oceans.
- Model foundations have improved through better understanding of the ocean (theory, observations, laboratory) and enhanced numerical methods.
- Computer power has increased to allow for refined resolution incorporating more details resulting in improved realism.

Models are the tool for large-scale ocean science

Numerical models are the *de facto* tool of choice for formulating and testing mechanistic descriptions of the ocean; for providing dynamically consistent state estimates; and forming the basis for dynamical predictions/projections.

Types of ocean and climate models



Compliments of Stephanie Waterman, University of British Columbia, Canada

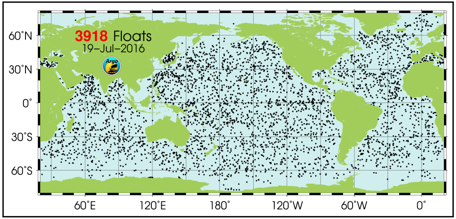
Some specific applications

- Mechanistic studies of ocean and climate processes
  - Process studies using fine resolution (≤ 1 km) simulations (MITgcm, SUNTANS)
  - Mechanisms for coastal and shelf processes (≤ 10 km) ((ROMS)
  - Mechanisms for observed large-scale variations (basin to global) (CLIVAR CORE)
  - Mechanisms for climate variability (basin to global) (MOM5, MOM6, NEMO)
- Operational predictions and state estimation
  - Coastal forecasting India INCOIS
  - Coastal forecasting USA NCEP
  - Ocean state estimation ECCO
- Projections for future climate change
  - IPCC-class simulations with anthropogenic forcing (CMIP)
  - Sea level changes (John Church's research group)
  - Changes in sea ice (NOAA info page)

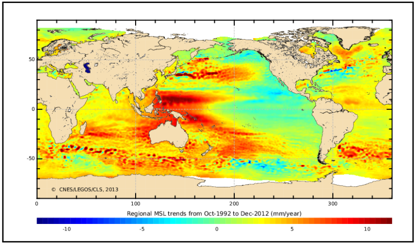
Revolution in ocean obs requires models to interpret

Near-global observations are pushing models to improve.

Argo + satellites provide high quality near-global information relevant for predictions and climate change. These and other data sources prompt modellers to use suitably configured and trusted models to facilitate full use and understanding of observation-based measures.



From Argo at UCSD



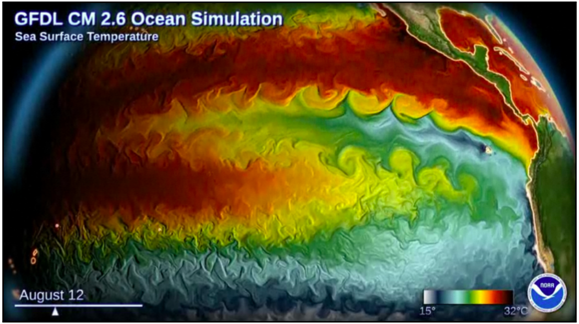
From AVISO

Outline

SST animation from GFDL CM2.6 climate model

Animation 1: Daily SST from the GFDL CM2.6

This coupled climate model uses a 0.1° configuration of MOM5 for the ocean component, under a 50 km global atmosphere model. It has been integrated for multiple-centuries in support of climate and ocean related studies. Available from Vimeo



Theoretical foundations for ocean models

- Continuum thermo-hydrodynamical equations of the ocean
  - Seawater mass conservation
  - Tracer mass conservation
  - Momentum conservation
  - Linear irreversible thermodynamics of seawater
  - Typically assume hydrostatic balance
- Boundary conditions
  - Air-sea interactions
  - Sea ice-ocean interactions
  - Ice shelf-ocean interactions
  - Solid-earth-ocean interactions
- Subgrid scale parameterizations
  - Momentum closure: frictional stress tensor
  - Tracer closure: transport tensor
  - Boundary layer parameterizations

Equations for ocean dynamics and thermodynamics

Thermo-hydrodynamic equations for the ocean

Momentum (Newton's 2nd Law)

$$\rho \, [\partial_t + (2\Omega + \omega) \times] \, v = -\rho \nabla (KE + GPE) + \nabla \cdot (\tau - 1p)$$

Mass conservation (continuity)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, v) = 0$$

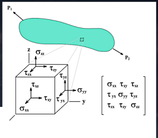
Enthalpy (heat) conservation)

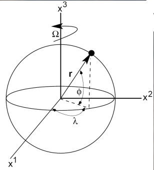
$$\frac{\partial (\rho \Theta)}{\partial t} + \nabla \cdot (\rho \, v \, \Theta + J_{\Theta}) = 0$$

Salt conservation

$$\frac{\partial (\rho S)}{\partial t} + \nabla \cdot (\rho \, v \, S + J_S) = 0$$

Equation of state relates density to temp, salinity, pressure $\rho = \rho(\Theta, S, p)$





Key dynamical speeds

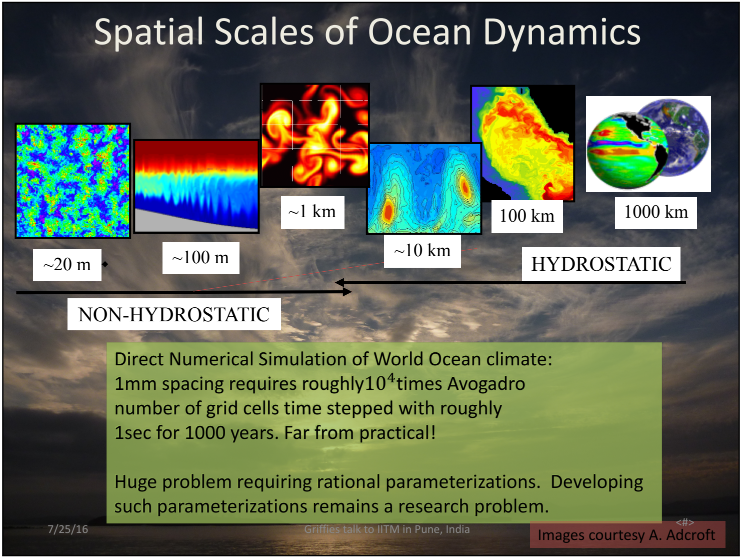
Acoustic: $C_s \sim 1500 \text{ m s}^{-1}$

External gravity: $\sqrt{gH} \sim 150 \text{ m s}^{-1}$

Internal gravity: $NH \sim 3 \text{ m s}^{-1}$

Advection: $U \sim 1 \text{ m s}^{-1}$

Spatial scales for ocean dynamical processes

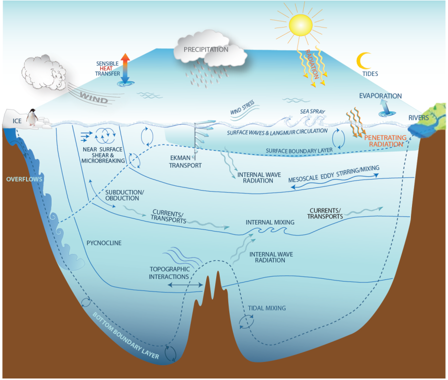


A sample of ocean processes

Slides in this section

- A zoo of physical ocean processes
- Space-time diagram of ocean motions
- Upper ocean boundary and wave interactions
- The marginal ice zone (MIZ)
- Southern Ocean processes
- The value of idealized Southern Ocean simulations
- Turbulent cascade of mechanical energy

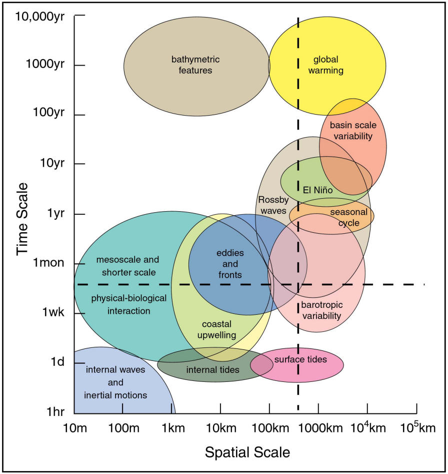
A zoo of physical ocean processes



From Griffies and Treguier (2013)

- The ocean contains a zoo of physical processes!
- Strong coupling between processes $\Leftrightarrow$ no spectral gap.
- Coupling means it is generally better to resolve than parameterize.
- Yet we cannot resolve everything $\Rightarrow$ a practical need for parameterizations that pass the "laugh test".

Space-time diagram of ocean dynamical processes



From Chelton (2001)

- Broad range of space-time scales
- We again see the absence of a clear spectral gap except for scales larger than 1000 km.

Upper ocean boundary and wave interactions

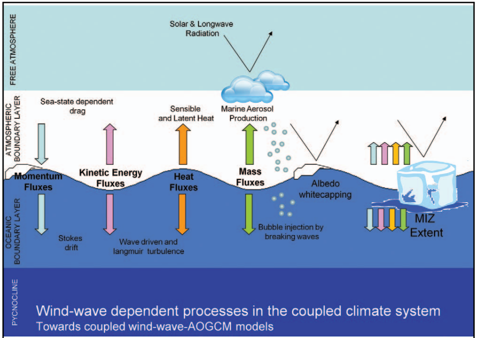
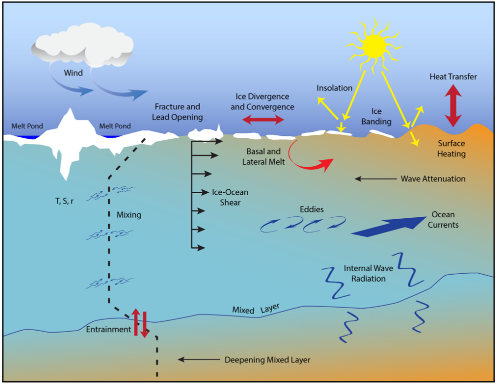


FIG. 1. A schematic view of the influence of waves on air–sea exchanges.

From Cavaleri et al (2012)

- New research activities in boundary layer param prompted by refined atmos and ocean resolutions that admit new dynamical regimes (e.g., mesoscale eddies, tropical cyclones).
- An increased awareness in the climate community of the importance of surface ocean gravity waves (e.g., Cavalieri et al (2012)).

The marginal ice zone (MIZ)



From ONR Marginal Ice Zonal Project

Questions about processes at the marginal ice zone are of prime importance as Arctic sea ice melts.

Southern Ocean processes

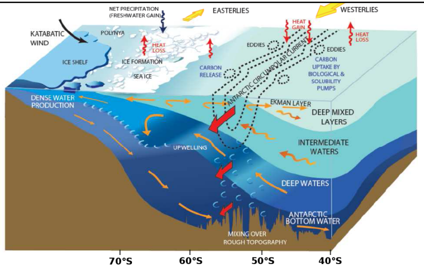


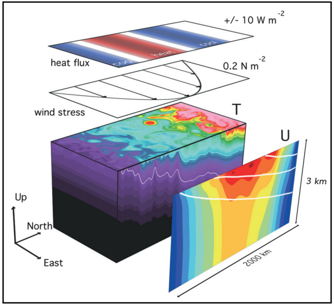
Fig. 1: Schematic of the Southern Ocean circulation, showing the numerous relevant physical and biogeochemical processes [courtesy of L. Talley].

- Fundamental role played by mesoscale eddies in transporting properties meridionally; absence of lateral boundaries make eddies dominate north-south transport.
- Eddy params are improving, but explicit resolution generally performs better.

Animation 2: SST in Southern Ocean from CM2.6

Daily mean SST in Southern Ocean, with animation thanks to Whit Anderson, GFDL.

A Southern Ocean process study



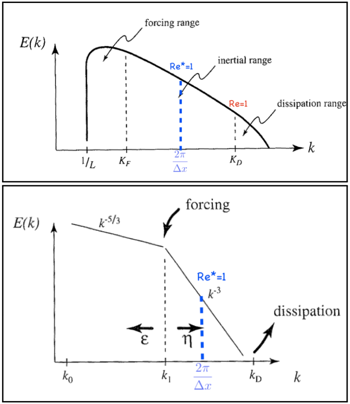
From Abernathey et al. (2011)

- The Southern Ocean is a region where mesoscale eddies are of leading order importance.
- This animation is part of an idealized study, and is shown here as an example how idealized process models can lend useful insight into the real ocean.

Animation 3: Southern Ocean channel

Animation from R. Abernathey, available from Vimeo

Turbulent cascade of mechanical energy



Compliments of Baylor Fox-Kemper, Brown University, USA

- 3d turbulence: energy cascade to small scales
- 2d/QG turbulence: energy cascade to large scales (inverse cascade)
- Cascades act to couple space-time scales.

Animation 4: QG turbulence cascade

Compliments of Shafer Smith, NYU USA

The ocean parameterization problem

Slides in this section

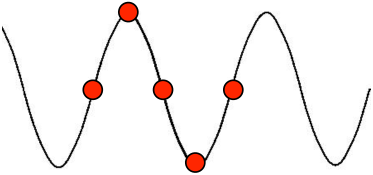
- Resolving versus parameterizing: some numbers
- Facets of what we mean by "resolution"
- Spatial scale of mesocale and submesoscale eddies
- Resolution required to represent mesoscale eddies
- Ocean resolution in IPCC-class climate models

Resolving versus parameterizing: some numbers

- Direct Numerical Simulation (DNS) of global ocean climate requires 3×10^{10} time steps of one second (1000 years).
- Setting the model's grid scale to the Kolmogorov length $\Delta = 10^{-3}$m over a global ocean domain of volume 1.3×10^{18} m^3 requires 1.3×10^{27} discrete grid cells. This is roughly $10^4 \times$ Avogadro's number!
- Each model grid point has a velocity vector and tracer fields to time integrate.
- Conclude:
    - We will be dust long before DNS of global ocean climate.
    - We must use parameterizations to simulate the ocean.
    - The rationalization of a DNS simulation typically requires a coarse-grained perspective, as certainly would DNS of the World Ocean climate.

Facets of what we mean by “resolution”

- general principles of resolution are the same for both atmospheric and ocean models
- there are different rules of thumb: one is that it takes 5 grid points to accurately define a feature without aliasing
- this means **1/8°** global resolution with an average horizontal grid cell of **14 km** can accurately depict only features larger than **56 km**
- models with variable grid spacing have variable resolution - beware of resolution-dependent physics!
- resolution is not cheap - because of the CFL* condition, as we shrink the horizontal grid spacing we must add vertical layers and decrease the time step

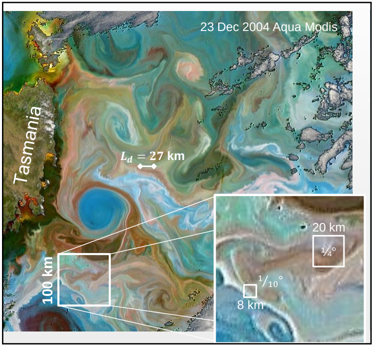


“every halving of the grid spacing requires roughly ten times as many computations”

* no transport faster than one grid cell per time step!

Compliments Stephanie Waterman, University of British Columbia, Canada

Spatial scale of mesocale and submesoscale eddies



MODIS satellite w/ inserts by A. Adcroft (GFDL)

- Eddy size $\propto$ first baroclinic Rossby Radius $\lambda_m = c_m/|f|$, where the phase speed is approximated by (Chelton et al. 1998)

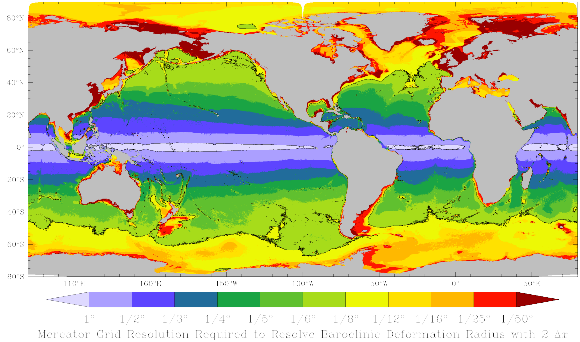
$$c_m \approx \frac{1}{m\,\pi} \int_{-H}^{0} N\,\mathrm{d}z.$$

- Global models are marginal at representing this scale; regional and process models just reach into the submesoscale.

Animation 5: Southern Ocean regional process model

MITgcm w/ $1/20^\circ$ (and $1/80^\circ$ local refinement) w/ 150 vertical levels. Available from YouTube

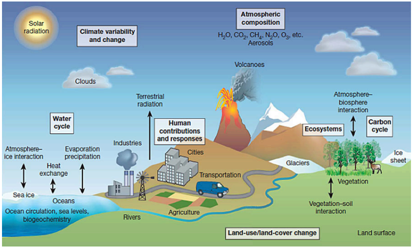
Resolution required to represent mesoscale eddies



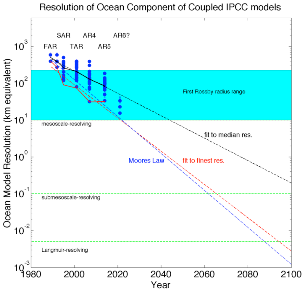
From Hallberg (2013)

- Hallberg (2013): $2\Delta \leq \lambda_1$ needed to resolve mesoscale eddies.
- Map indicates the necessary Mercator spacing for $2\Delta = \lambda_1$.

Ocean resolution in IPCC-class climate models



Compliments of GFDL



From B. Fox-Kemper, Brown University, USA

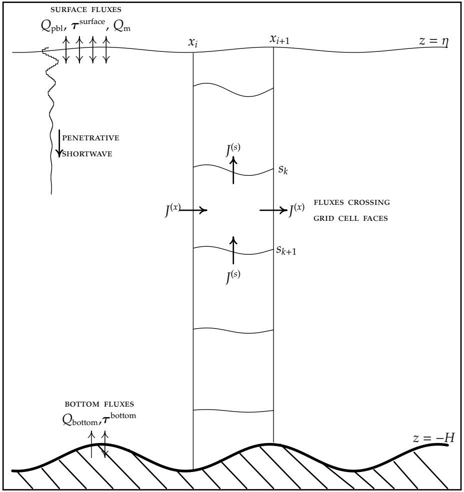
- The ocean is but one component amongst many within climate system models.
- Resolution refinement is painfully slow!
- This diagram is useful to target one's career choices.

Partitioning the vertical

Slides in this section

- Discretizing a column of ocean fluid
- Vertical coordinate representation
- Geopotential or pressure vertical coordinates
- Isopyncal vertical coordinates
- Sigma or terrain following vertical coordinates

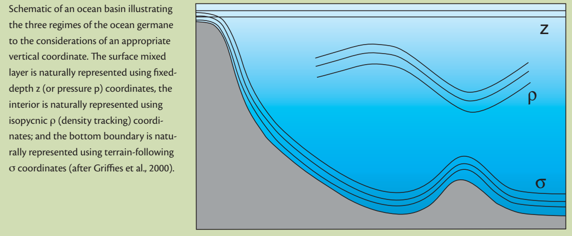
Discretizing a column of ocean fluid



From Griffies and Treguier (2013)

- Boundary fluxes through surface and bottom.
- Transport convergence (advective and subgrid scale), body forces (gravity, Coriolis), contact forces (pressure, friction), and penetrative radiation render time tendency for mass, tracer, and momentum.
- Generally fix the horizontal position of grid cells, but allow for upper and lower interfaces to be functions of time (e.g., z^*, pressure, σ, isopycnal)

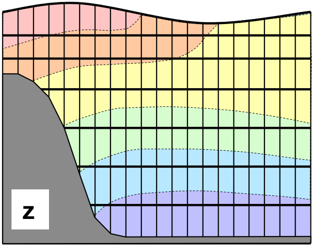
Vertical coordinate representation



Adapted by Chassignet et al (2006) from original figure in Griffies et al (2000)

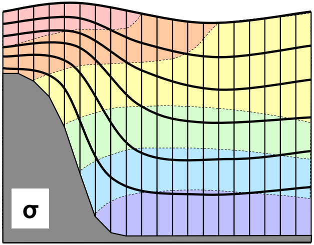
- GEOPOTENTIAL OR PRESSURE: common for non-hydrostatic process modelling and large-scale climate modelling (MITgcm, MOM, NEMO)
- ISOPYCNAL: clean representation of interior quasi-adiabatic flows and overflows (GOLD, HYCOM)
- SIGMA OR TERRAIN FOLLOWING: common for shelf/coastal modelling ((ROMS) and Curchitser's lectures)

Geopotential or pressure vertical coordinates



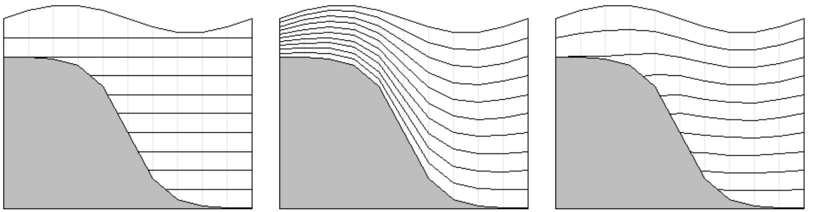
- Most common method for global models; extensive experience.
- Generalizations: $z^* = H\,(z - \eta)/(H + \eta)$ absorbs SSH undulations; pressure (mass conserving).
- Spurious diapycnal mixing if poorly chosen numerical methods & parameter settings (e.g., Ilicak et al 2012).
- Downslope flows poorly represented absent very fine resolution (Winton et al. 1998).

Sigma or terrain following vertical coordinates



- Extensive applications for coasts & shelves
- Traditionally $\sigma = (z - \eta)/(H + \eta)$, but with generalizations.
- As for geopotential, $\exists$ spurious diapycnal mixing with poorly chosen numerical methods & parameter settings.
- Much care is needed to handle horizontal pressure gradient calculation; generally need to smooth topography.
- There are very few global climate realizations.

Undulations of coordinate surfaces w/ z, σ, and z^*

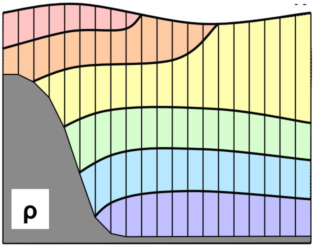


z σ z^*

Animation 6: undulations of coordinate surfaces

Animation to illustrate undulating z, σ, and z^* coordinate surfaces in the presence of a gravity wave. Compliments A. Adcroft (GFDL).

Isopyncal vertical coordinates



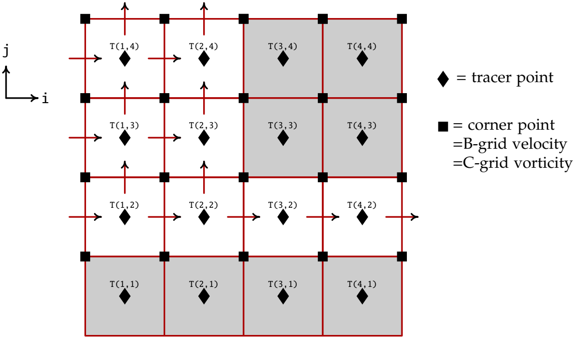
- Quasi-adiabatic interior & flow-topography interactions (e.g., overflows)
- Inherently poor representation if weak vertical stratification (e.g., Labrador Sea, Southern Ocean, coastal regions).
- Care needed to represent realistic ocean thermodynamics and conservative transport, though proven methods now common.
- GFDL-GOLD, HYCOM, and Bergen

Partitioning the horizontal

Slides in this section

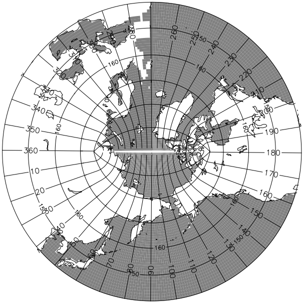
Horizontal representation: structured finite volume

Tracer cells T(i,j) with fluxes and land/sea masking

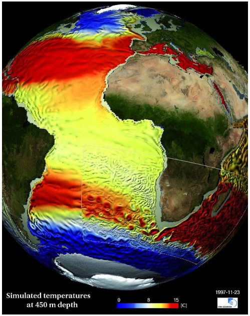


- Most common approach since 1960s; e,g., HYCOM, MITgcm, MOM, NEMO, ROMS.
- Recent advances with nesting allow for multi-scale simulations (Debreu and Blayo, 2008)

Examples of structured finite volume grids



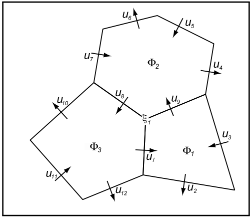
From Griffies et al (2005)



From Biastoch et al (2009)

- Tripolar common for global models
- Nested example has refined grid (0.1°) around South Africa embedded in global grid (0.5°) to examine Agulhas eddies impact on Atlantic circulation.

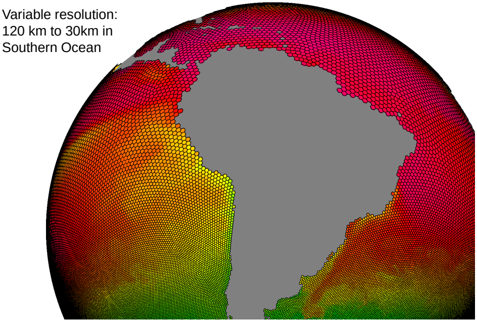
Horizontal representation: unstructured finite volume



- Aimed at seamless representation of multiple-scales.
- Can obtain enhanced conservation properties in discrete equations.
- Indirect addressing of algorithms adds computational cost (i.e., number of neighbors unknown *a priori*).
- Los Alamos and NCAR have a maturing effort: MPAS-ocean (Ringler et al 2013).
- Nascent effort ongoing at MPI-ICON

Example of unstructured finite volume grid

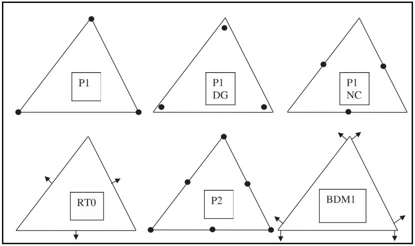
MPAS Horizontally Unstructured Grids



Compliments of M. Petersen, LANL (2013)

- Refined resolution towards Southern Ocean.
- Note: this is not from a working model; it is a mere example of the grid capabilities.

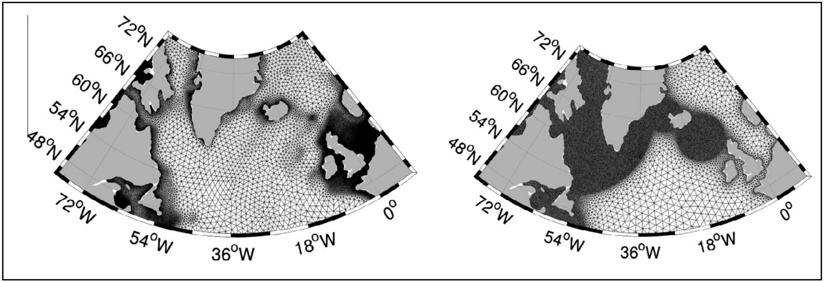
Horizontal representation: unstructured finite element



From Danilov (2013)

- Aimed at seamless representation of multiple-scales.
- Decompose continuous equations using basis functions and matrix inversions.
- Indirect addressing adds computational cost.
- Effort at AWI focused on climate: FESOM
- Effort at Louvain focused on shallow ocean: SLIM
- Non-hydrostatic process model at Stanford: SUNTANS

Examples of unstructured finite element meshes



From Danilov (2013)

- Example 1: refined resolution in coastal regions
- Example 2: refined resolution in regions of dynamical interest

Outline

4 Some perspectives

Frontier issues I

Personal reflections on where the envelope is being pushed

1. Role of mesoscale eddies in climate
   - Global models are increasingly being run with an explicit representation (albeit imperfect) of the mesoscales.
   - How/will climate sensitivity, variability, predictability be modified with eddying ocean simulations?
   - How well do parameterized models emulate eddying models?
2. Parameterizations, including stochastic methods
   - Although many modelling centres can now run eddying simulations, we will need mesoscale eddy parameterizations for many decades.
   - Stochastic methods are being successfully used for atmospheric parameterizations. They are being used in ocean parameterizations as well.

Animation 7: Southern Ocean in MOM5 at 5km

Compliments of Australian Center of Excellence for Climate System Science (A. Hogg, P. Spence, M. England) Available from YouTube.

Frontiers issues II

More reflections on where the envelope is being pushed

1. Multi-scale modelling
   - Whether structured (with nesting) or unstructured, models are being applied to address problems with multiple scales.
   - Impacts of coast on large-scale, and converse.
   - Seamless modelling is a dream that is being pushed for scientific and non-scientific reasons.
2. Coupling circulation models to surface wave models
   - As ocean and atmospheric models refine resolution, traditional methods for parameterizing air-sea interface start to break down; e.g., Monin-Obukov similarity theory shows its limitations.
   - The upper ocean exhibits waves, and waves affect the coupling.

Animation 8: ocean surface wave model Wavewatch III

Ocean surface waves affect air-sea interactions as well as upper ocean mixing. Animation compliments of Yalin Fan, Stennis Space Center, USA.

Three elements of ocean modelling

The three elements to questions of ocean modelling

1. Model fundamentals (science of ocean models)
   - math/physics formulations
   - parameterizations
   - numerical methods
2. Boundary forcing/fluxes (the ocean in the climate system)
   - The ocean is a forced-dissipative system.
   - Boundary fluxes are poorly measured in the real world, so poorly constrained in models.
3. Analysis methods (articulating the science from simulations)
   - The rationalization and communication of simulation results is just as important as the rationalization of the simulation setup.
   - Theories and equations used to formulate the model equations form the starting point for analyzing the output.

Trust but verify

Models are most useful when appreciating their limitations.

- Model limitations arise from:
  - fundamentals and/or numerical methods;
  - configuration design;
  - boundary information and/or other component models;
  - computational power.
- Do not treat models as a black box.
  - Models are tools to help deduce mechanisms.
  - Use diagnostic methods to uncover reasons for particular behaviour.
- Numerical errors often appear in physically interesting manners.
  - Remain skeptical even if the simulation "looks right".
  - Investigate why and how.
  - The more one learns about models, the more one can sense whether a particular simulation is physically sensible or the artefact of faulty methods.

Wherein lies the truth?

Physical “laws” are not determined by political processes.

- Science rests on the shoulders of those before us. Sometimes those shoulders are strong; sometimes they need strengthening or toppling.
- Respect those before us, but also question their scientific story.
- Exercise a balance between trust and verification.

Perspectives on models

- Models cannot be *validated*. At best, they can be *evaluated* (see Oreskes et al 1994).
- “All models are wrong, but some are useful.” (Wikipedia on G.E.P. Box, statistician)
  - An ocean model is a rather tarnished reflection of the real ocean, allowing experimental investigation and offering quantitative predictions.

Closing comments from a fellow Indian

> *Do not believe in anything simply because you have heard it... Do not believe in anything merely on the authority of your teachers and elders... But after observation and analysis, when you find that anything agrees with reason and is conducive to the good and benefit of one and all, then accept it and live up to it.*
>
> Gautama Buddha

Closing comments from a fellow Indian

Do not believe in anything simply because you have heard it... Do not believe in anything merely on the authority of your teachers and elders... But after observation and analysis, when you find that anything agrees with reason and is conducive to the good and benefit of one and all, then accept it and live up to it.

Gautama Buddha

Outline

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